

## CONSTITUTIVE MODELING FOR ISOTROPIC MATERIALS\*

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## INTRODUCTION

The objective of the present program is to develop a unified constitutive model for finite-element structural analysis of turbine engine hot section components. This effort constitutes a different approach for non-linear finite-element computer codes which have heretofore been based on classical inelastic methods. The unified constitutive theory to be developed will avoid the simplifying assumptions of classical theory and should more accurately represent the behavior of superalloy materials under cyclic loading conditions and high-temperature environments. This class of constitutive theory is characterized by the use of kinetic equations and internal variables with appropriate evolution equations for treating all aspects of inelastic deformation including plasticity, creep, and stress relaxation. Model development is directed toward isotropic, cast nickel-base alloys used for air-cooled turbine blades and vanes. Recent studies have shown that this approach is particularly suited for determining the cyclic behavior of superalloy-type blade and vane materials and is entirely compatible with three-dimensional inelastic finite-element formulations. More efficient and accurate inelastic analysis of hot section components--turbine blades, turbine vanes, combustor liners, and seals--fabricated from "age hardenable" isotropic superalloy materials will be realized as the result of these developments.

During the first two years of the program, extensive experimental correlations have been made with two representative unified models. The experiments are both uniaxial and biaxial at temperatures up to 1093°C (2000°F). In addition, the unified models have been adapted to the MARC finite element structural code and used for stress analysis of notched bar and turbine blade geometries.

## CONSTITUTIVE MODEL SELECTION

A literature survey was conducted to assess the state-of-the-art of time-temperature dependent elastic-viscoplastic constitutive theories which are based on the unified approach. As reported earlier (ref. 1), the review identified more than ten such unified theories which are shown to satisfy the uniqueness and stability criteria imposed by Drucker's postulate and Ponter's inequalities. These theories are compared on the basis of the types of flow law, kinetic equation, evolution equation of the internal variables, and treatment of temperature dependence.

As a result of the literature survey, the models of Bodner and Partom (ref. 2) and of Walker (ref. 3) were selected for further study. These two models are representative of the class of unified models considered in the review process but differ significantly in the choice of particular functional forms for the basic flow law, the kinetic relationship, the parameter used as a measure of hardening, and the evolution equations for the internal variables describing work hardening. Thus, a direct comparison between these two models and the experiments should illustrate well the consequences of a wide range in constitutive modeling approach. It

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is also significant that both models have already found significant application to analysis of gas turbine materials and to hot section components. Therefore, they are further along in their development and evaluation than most of the other comparable models.

## MATERIAL SELECTION

Two materials widely used in gas turbines are being evaluated with the constitutive models: PWA alloy B1900+Hf and MAR-M247. Tests completed to date have been with B1900+Hf. Casting configurations and pour and mold temperatures were selected to assure a grain size of ASTM No. 1 to 2 in the gage section and a  $\gamma'$  size of 0.9  $\mu\text{m}$  in the fully heat treated condition for the B1900+Hf.

## EVALUATION OF UNIFIED MODELS

The testing program included uniaxial and biaxial (tension-torsion) isothermal specimens in monotonic, creep, stress relaxation, and cyclic loading sequences encompassing wide variations in strain rate and temperature. Various thermo-mechanical fatigue cycles were investigated also. In each case, the measured material response was correlated with the two unified constitutive models. A small subset of the data was used to determine the constants for each model; about eight to ten constants are required. Also, definitive procedures were developed to determine each constant. The data correlations presented are based on the models with material constants determined from tensile or tensile and a few fatigue cycles. In this sense the results are predictive.

Some representative results are shown in Figures 1-4 for tensile, creep, cyclic, and TMF load histories. Over 60 different loading histories were examined with generally good correlation using both models.

Correlation of local deformation at the root of a notched round tensile specimen was not as good as for the homogeneously-stressed test specimens. This was attributed to specimen material inhomogeneity and to accuracy of the material model in the near yield (small inelastic strain) region.

## CONCLUSIONS

The results of this program provide strong evidence for the applicability of the unified constitutive equation approach to describe the strongly non-linear, time- and temperature-dependent response of metals to arbitrary load or deformation histories. As a minimum, the two models studied were correlative in that they demonstrated reasonable agreement with all the experimental data generated over a very wide range of temperature, deformation rate, and load cycle. This was accomplished with a fixed model and fixed set of material constants. In a broader sense, the models were predictive in that the functional representations and the material constants were obtained from a small subset of the total tests performed. Even better overall correlation could have been achieved if all the data were used to optimize the functions and constants in each model.

A major criticism of the unified models has been the apparent (and often real) difficulty in defining and experimentally determining the multiple material constants

employed in the models. We believe we have made significant progress in this regard. The different terms in the constitutive equations are very interactive so that it is essential to have a physical insight into the meaning of each function and its associated constants. Procedures developed delineate a deterministic method for evaluating the constitutive constants in a sequential manner from a relatively small material test data base.

While the correlative and predictive capacity of the unified models has been demonstrated, their practical application will depend on their adaptability in efficient computational algorithms when incorporated into finite element or other numerical methods for structural analysis. Preliminary use with the MARC finite element code indicates that improvement is needed in the numerical integration procedures used with these stiff, nonlinear equations. The computation time for an equivalent problem is approximately the same order of magnitude as when using classical, time-independent plasticity formulations.

### REFERENCES

1. Lindholm, U. S.; Chan, K. S.; Bodner, S. R.; Weber, R. M.; Walker, K. P.; and Cassenti, B. N.: Constitutive Modeling for Isotropic Materials. Annual Report NASA CR-174718, 1984.
2. Bodner, S. R.; and Partom, Y.: ASME J. of Applied Mechanics, vol. 42, 1975, p. 385.
3. Walker, K. P.; NASA Contract Report NASA CR-165533, 1981.

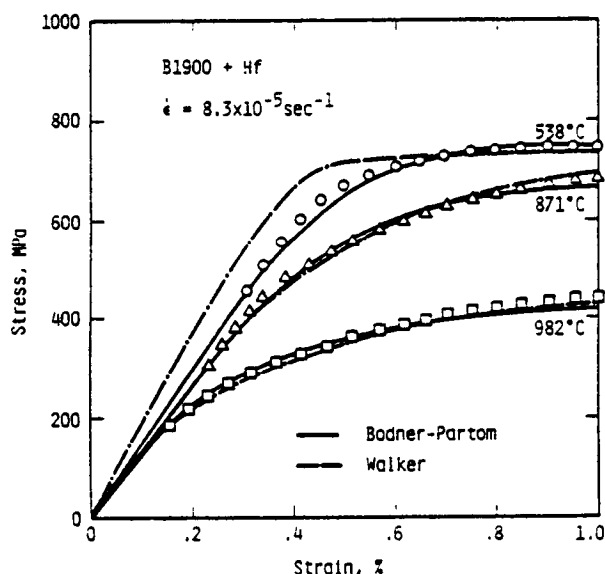


Figure 1

COMPARISON OF EXPERIMENTAL AND CALCULATED STRESS-STRAIN CURVES OF B1900+Hf AT THREE TEMPERATURES

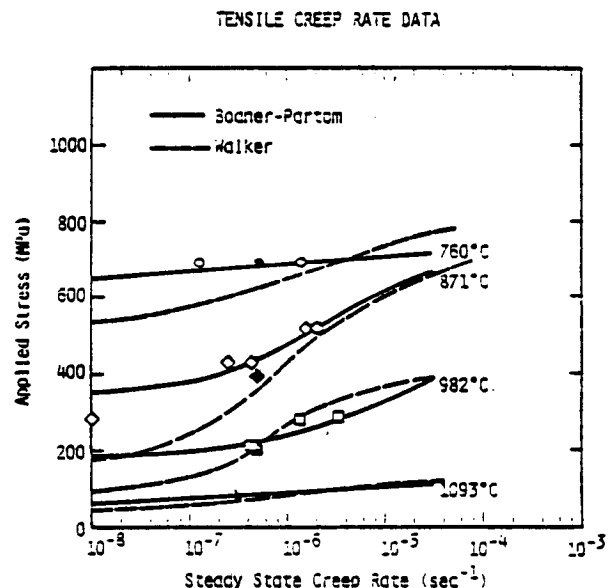


Figure 2

THE CALCULATED AND EXPERIMENTAL RESULTS OF CONSTANT LOAD CREEP TESTS

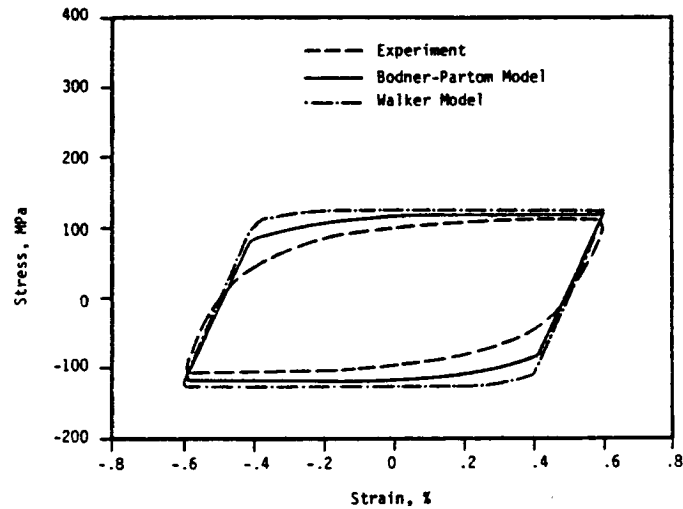


Figure 3

COMPARISON OF THE EXPERIMENTAL AND CALCULATED  
STABLE HYSTERESIS LOOPS OF B1900+Hf at 1093°C

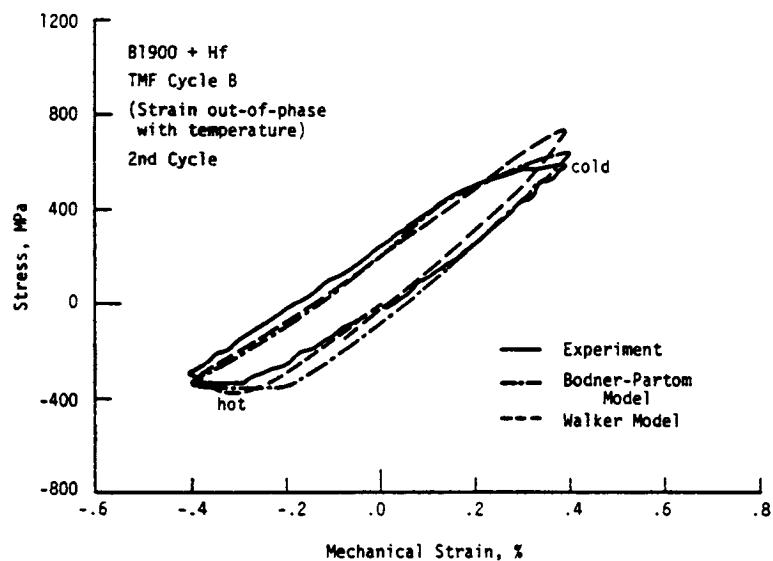


Figure 4

THERMOMECHANICAL CYCLIC DATA COMPARED WITH THE  
WALKER AND THE BODNER-PARTOM MODEL PREDICTION  
FOR THE OUT-OF-PHASE TMF CYCLE